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REPLY

Further 'comment on 'Generalized Bessel functions in tunnelling ionization''

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Abstract
J Bauer, in commenting on our tunnelling approximation for the generalized Bessel function, points out that when the approximation is applied to strong-field ionization, it is suitable only for the lowest-energy part of an ionization spectrum. We do not disagree. We point out several things: the results of Bauer are to be expected; linear polarization results are dominated by the lowest part of the multiphoton spectrum; and we do not recommend practical use of this tunnelling approximation, since the asymptotic approximation is so much better. We show comparisons of momentum distributions calculated with the tunnelling approximation and those with the complete strong-field approximation, which show in more detail than spectrum comparisons that the tunnelling approximation to the generalized Bessel function is applicable only to the low-momentum part of the distribution, and neglects altogether the high-momentum portion.

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In a recent paper [1], we showed how one can derive explicitly a tunnelling form of the generalized Bessel function, $J_n(u, v)$, that arises in theories of strong-field-induced processes. In his comment [2], Bauer points out that poor results may be obtained for the description of spectra using the tunnelling approximation, and that total ionization rates will be systematically understated. We agree with this appraisal, although we regard this as self-evident. The fully-stated SFA (strong-field approximation) contains momentum components with a broad range of values, whereas the tunnelling approximation specifically limits possible momenta because of the $u \ll |v|$ requirement. Another way to state this is to note that the SFA can be regarded as a sum of tunnelling and multiphoton contributions, and only the tunnelling part is retained in our approximation.

The result that the high end of photoelectron spectra is lost in our tunnelling approximation is a direct and anticipated consequence of the limitation to low-energy electrons as a way of

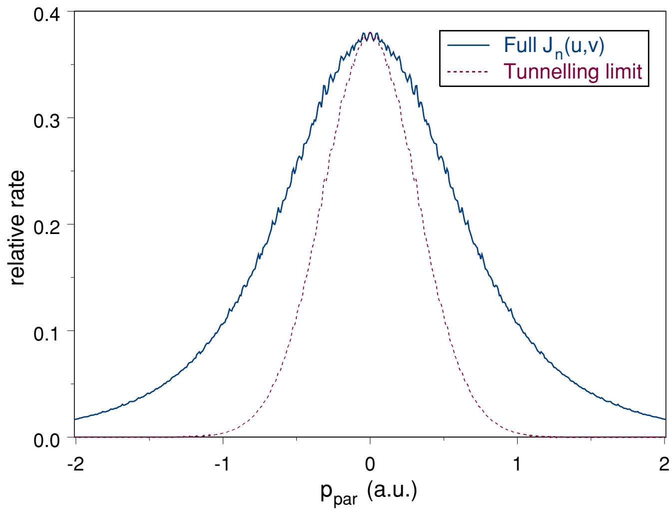

Figure 1. Momentum distribution of photoelectrons from ionization of 1s hydrogen by light of 800 nm, as in an example given by Bauer [2]. Field intensity is such that $z_1 = 40$, where $z_1 \equiv 2U_p/E_B$, U_p is ponderomotive energy, and E_B is atomic binding energy. The distribution is with respect to the momentum component parallel to the polarization direction of the laser. The tunnelling limit and the full $J_n(u, v)$ coincide only for small momenta. Relative rates are calculated by the SFA assuming a Gaussian time distribution in a laser pulse of 25 fs.

extracting the tunnelling limit. The fact that this happens follows immediately from the property that our approximation to arrive at the tunnelling result effectively discards the high-energy part of the spectrum. Total rates with the tunnelling limit will necessarily always be less than those with the full $J_n(u, v)$. The degree to which this property affects total rates depends strongly on the atom and on field parameters.

It is necessary to emphasize that the SFA [3] is *not*, *ab initio*, a tunnelling theory. This is often overlooked because the 'KFR' (Keldysh [4], Faisal [5], Reiss [3]) designation lumps the SFA together with the Keldysh method. The SFA is always expressed in terms of an index (the 'n' in $J_n(u, v)$) that plays the role of photon order, with tunnelling following only as a particular limit; whereas the Keldysh method is an *a priori* tunnelling approximation. In this connection, we need to emphasize that specific tunnelling theories such as PPT [6] (Perelomov, Popov, Terent'ev) and ADK [7] (Ammosov, Delone, Krainov) are unrelated to the tunnelling limit of the SFA. To reach the tunnelling limit of the SFA, one must discard part of the overall SFA result, whereas explicit tunnelling theories are complete theories constructed around the tunnelling hypothesis.

It is instructive to compare the tunnelling limit of the SFA to the complete SFA in terms of momentum distributions rather than energy ($p^2/2$) spectra, since momentum distributions emphasize the low end of the spectrum where one expects agreement. Figure 1 shows the distribution of momenta $p_{\shortparallel}$ measured in a direction parallel to the polarization vector of the field for the first example of hydrogen 1s ionization treated by Bauer. One can see clearly the detailed correspondence for low momenta, as well as the fact that the tunnelling approximation for $J_n(u, v)$ explicitly lacks the large-momentum components. A more extreme case of this behaviour is shown in figure 2, for single ionization of neon, where the momentum distribution has a minimum at $p_{\shortparallel} = 0$. Because the tunnelling portion of the ionization for neon under the stated conditions is small as compared to the overall momentum distribution, there will be a major difference in total rates, much greater than for the example in figure 1.

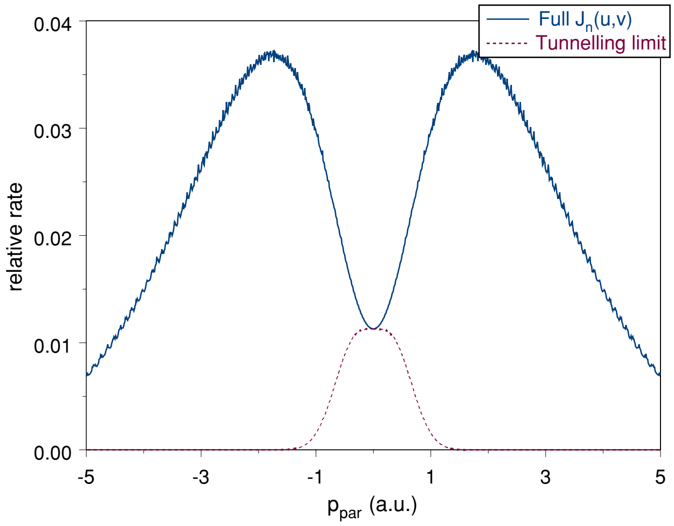

Figure 2. This momentum distribution is comparable to figure 1 except that it is for the case of neon, which has a dip in the momentum distribution near $p_{\shortparallel} = 0$. Wavelength and z_1 values are the same as for figure 1. The result of the dip is that most of the transition rate comes from values of the momenta that are neglected in the tunnelling approximation to $J_n(u, v)$.

To summarize, we concur with Bauer that it is far better for SFA calculations to employ the asymptotic approximation [3] to $J_n(u, v)$ rather than the tunnelling approximation. The asymptotic form of $J_n(u, v)$ is an explicit algebraic expression that computes quickly, despite its complicated algebraic form. It is an excellent approximation to $J_n(u, v)$ whenever the frequency is low, and it avoids the lengthy and error-prone complete computation of $J_n(u, v)$ that requires summation of products of ordinary Bessel functions. Nevertheless, we have shown [1] in detail how one can express the tunnelling limit of the $J_n(u, v)$ function that arises in the complete statement of the SFA. The connection is plainly shown in figures 1 and 2. Furthermore, our figures show the underlying reason for discrepancies in total rate calculations: the SFA contains a part that can be ascribed to tunnelling, plus a part that one could describe as 'multiphoton'. (We regard the tunnelling-multiphoton distinction as misleading, not least because it will vary with calculational method.)

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